A NOVEL VOLUME CT WITH X-RAY ON A TROUGH-LIKE SURFACE AND POINT DETECTORS ON CIRCLE-PLUS-ARC CURVE

H. Xu, and T.G. Zhuang
Department of Biomedical Engineering, Shanghai Jiaotong University, Shanghai, P. R. China

***Abstract* - A novel imaging mode of cone-beam volume CT is proposed in this paper. It adopts a raster scanning x-ray source on a trough-like surface, and a group of point detectors distributing on a large circle plus an orthogonal arc. Through a single rotation of the trough-like surface, a full set of projection data can be acquired and an accurate 3D image can be reconstructed. This paper describes the shape and dimension of the trough-like x-ray source and the configuration of detectors in detail, proves that this structure is satisfied with the completeness condition, and gives a reconstruction algorithm of 3-D image adapting to this structure. Computer simulations show that this structure could achieve optimal result in meeting the following requirements: minimum time of rotation, less Compton scatter, and fast image reconstruction speed while keeping the completeness condition to be realized.**
***Keywords* - Volume CT, completeness condition, trough-like surface, point detector, image reconstruction**

I. INTRODUCTION

As an extension technology of traditional computed tomography, direct volume CT has been a hot topic in recent years. Comparing with traditional CT, volume CT improves the dose usage of X-ray, the speed of projections collecting, and most important it is able to provide isotropic reconstruction images in spatial domain.

To reconstruct a 3-D image exactly from 2-D projection data, Tuy [1] showed that each projection plane passing through the object should intersect the orbit of the source vertex. This requirement is referred to as Tuy's data completeness condition. Only if the completeness condition is met, the precise reconstruction could be ensured.

At present, there are several methods for direct volume CT, such as direct volume CT based on ideal vertexes of the cone-beam [2], direct volume CT based on limited projection [3], helical or spiral volume CT [4]. These scanning modes of cone-beam CT have the following shortcomings: (1)the ideal non-planar geometry of the vertex orbits are almost impossible to be implemented, (2)the effect of Compton scatter to planar detectors is so heavy that reconstruction images have severe artifacts, (3)the reconstruction algorithm is very complex and time-consuming, and so on.

In this paper we break all above conventional scanning structures of volume CT, and propose a new imaging mode of volume CT based on a trough-like x-ray source and point detectors. This structure could gain full projections through a single time rotation of the trough-like x-ray source.

II. NEW IMAGING MODE OF VOLUME CT

A. Reverse Geometry of X-ray Radiology

R. D. Albert, T. M. Albert proposed a reverse geometry of x-ray (RGX) technology [5]. The principle of the RGX system is that the object is placed next to a large raster scanning x-ray source with a point detector located 5cm~3m away from the object.

The remoteness and small size of the detector eliminates nearly all scattered x-rays from detection, therefore it would improve resolution and avoid artifacts in the reconstructed images.

B. A Direct Volume CT with X-ray from a Trough-like Surface and Point-detectors on Circle-plus-arc Curve

We adopt RGX mode in direct volume CT and mend the structure of RGX in following ways. Its scanning structure is shown in Figure 1. Firstly, to gain complete projections, we use a trough-like surface instead of a planar raster scanning x-ray source. Secondly, in stead of a single point detector, we use a group of point detectors distributed on a large circle plus a perpendicular arc in the gantry to collect projections,.

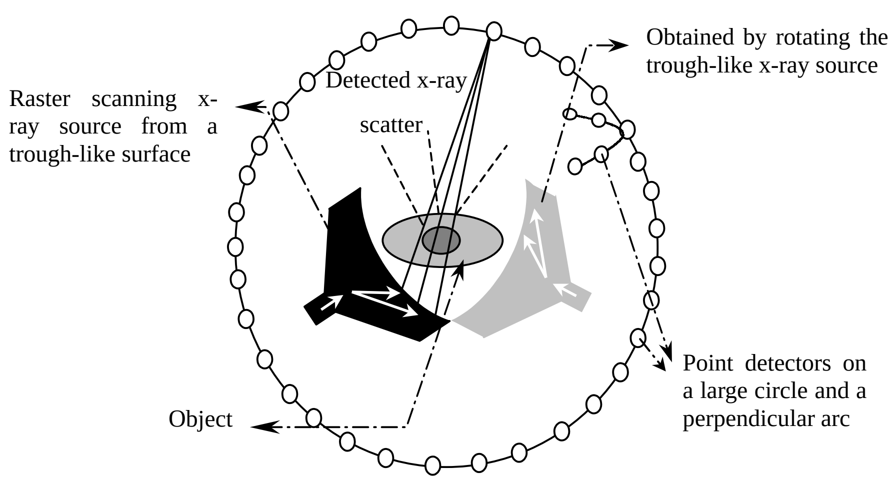


Figure 1. The structure of a novel volume CT with x-ray from a trough-like surface and point-detectors on circle-plus-arc curve.

Report Documentation Page

Report Date	Report Type	Dates Covered (from... to)
25 Oct 2001	N/A	-

Title and Subtitle A Novel Volume CT With X-Ray on a Trough-Like Surface and Point Detectors on Circle-Plus-ARC Curve	**Contract Number**
	Grant Number
	Program Element Number
Author(s)	**Project Number**
	Task Number
	Work Unit Number
Performing Organization Name(s) and Address(es) Department of Biomedical Engineering Shanghai Jiaotong University Shanghai, P.R. China	**Performing Organization Report Number**
Sponsoring/Monitoring Agency Name(s) and Address(es) US Army Research, Development & Standardization Group (UK) PSC 802 Box 15 FPO AE 09499-1500	**Sponsor/Monitor's Acronym(s)**
	Sponsor/Monitor's Report Number(s)
Distribution/Availability Statement Approved for public release, distribution unlimited	
Supplementary Notes Papers from 23rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society, October 25-28, 2001, held in Istanbul, Turkey. See also ADM001351 for entire conference on cd-rom. , The original document contains color images.	
Abstract	
Subject Terms	
Report Classification unclassified	**Classification of this page** unclassified
Classification of Abstract unclassified	**Limitation of Abstract** UU
Number of Pages 4	

The point detectors which evenly distribute on a large circle and an arc are indicated by open circles in Figure 1. The trough-like x-ray source at the initial position is to be rotated to the second position as indicated by the gray shape in the figure to ensure the completeness of projections. To simplify the mathematical derivation, it is assumed that the object, the x-ray source orbit and the detectors are concentric circles with the center at point O, and radii r_1, r_2, r_3 respectively. They should meet certain conditions according to Tuy's completeness condition.

According to the completeness condition, x-ray from the trough-like surface should not be sheltered by itself. Through calculating, we have a set of approximate solutions: $r_2 = 3r_1$, $r_3 = 8r_1$.

Now let's show the length of the perpendicular arc, the other part of detectors orbits.

To satisfy the completeness condition, and in terms of the above set of approximate solution, we set the radian of perpendicular arc to $4\arcsin\frac{1}{8}$.

Furthermore, we need to know the width of the trough-like surface. Through the extreme point of the perpendicular arc we draw a tangent line of the object. The point where the tangent line intersects the trough-like surface is the extreme point of the trough. After calculating, we set its width to $3r_1$.

C. The Proof of the Completeness Condition

Through a single movement of the trough-like x-ray source, the volume imaging system described in this paper could "wrap" the object completely. As is known that any plane through the object intersects the orbits of the cone-beam x-ray at least once. So the system satisfies Tuy's completeness condition.

Moreover, from the view of the detectors, they evenly distribute on a large circle and a perpendicular arc, which is equivalent to that the vertex of a cone-beam x-ray moves on the orthogonal circle-and-arc orbit. This shows that the scanning structure satisfies completeness condition. We have to point out that the detectors at the lower part of the large circle can't gain all projections for sheltered by the trough-like surface itself. But the radian of perpendicular arc is $4\arcsin\frac{1}{8}$, then the detectors on the perpendicular arc can gain the projections which are sheltered. From all of above, the direct volume CT system satisfies Tuy's completeness condition.

III. Image Reconstruction

A. Cone-beam Projection and 3-D Radon Transform

3D Radon transform and Grangeat's formula are the typical and efficient reconstruction algorithm for volume CT based on orthogonal circle-and-arc orbit.

The cone-beam projection $g(S,A)$ of the object function $f(\vec{x})$ can be expressed as

$$g(S,A) = \int_{-\infty}^{+\infty} f(\overrightarrow{OS} + t\hat{\sigma})dt \qquad (1)$$

where $g(S,A)$ is the projection from the cone-beam focal point S to a point A in the detector plane.

The Radon transform of the object is defined as the plane integral of $f(\vec{x})$. In Figure 2, any Radon plane ζ can be defined by a unit vector $\hat{\theta}$ and a scalar ρ where

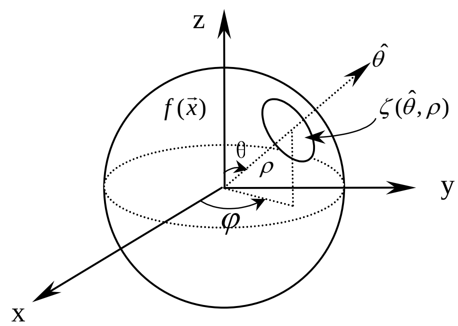


Figure 2. The Radon transform is the plane integral of the object $f(\vec{x})$.

$$\hat{\theta} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \qquad (2)$$

is the normal vector of plane $\zeta(\hat{\theta},\rho)$ and ρ is the distance from this plane to the origin of the coordinate system. The Radon transform of the object $f(\vec{x})$ is given by

$$R(\hat{\theta},\rho) = \iiint f(\vec{x})\delta(\vec{x}\cdot\hat{\theta} - \rho)d\vec{x} \qquad (3)$$

where the δ function constrains the 3D integral within plane $\zeta(\hat{\theta},\rho)$.

So the object function $f(\vec{x})$ can be exactly reconstructed by using the inverse 3D Radon transform [6]

$$f(\vec{x}) = -\frac{1}{4\pi^2}\iint_{2\pi}\left[\frac{\partial^2}{\partial\rho^2}R(\hat{\theta},\rho)\right]\Bigg|_{\rho=\vec{x}\cdot\hat{\theta}} d\omega \qquad (4)$$

where $d\omega = \sin\theta d\theta d\varphi$. $f(\vec{x})$ can be reconstructed, if we could gain $R'(\hat{\theta},\rho) = \frac{\partial}{\partial\rho}R(\hat{\theta},\rho)$ from the projection data $g(S,A)$. Grangeat's formula can complete this conversion process.

B. Grangeat's Formula

Grangeat's formula relates the cone-beam projection of $f(\vec{x})$ with the first derivative of its 3D Radon transform.

In the local coordinate system (μ,υ,ω) of Figure 3, the virtual detector plane ξ is defined by the υ axis and ω axis. The μ axis is coincident with $\overrightarrow{OS}$. The Radon plane $\zeta(\hat{\theta},\rho)$, where the plane integral takes place, goes through the focal point S and intersects the detector plane ξ at line D_1D_2. Another local coordinate system (μ,p,q) is defined with the rotation of the υ axis and ω axis about the μ axis by an angle α ($\alpha\in[0,\pi)$),

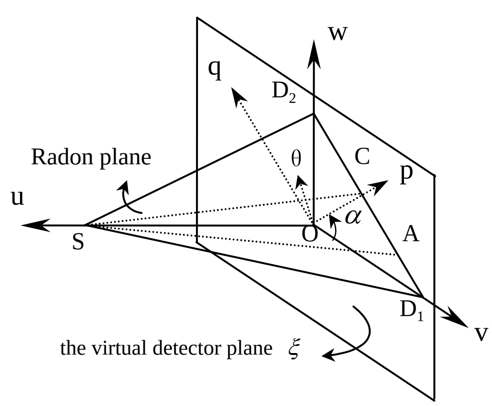


Figure 3. The coordinate system of cone-beam projection to establish Grangeat's formula.

such that the p axis is perpendicular to line D_1D_2. The distance from the origin of the coordinates to line D_1D_2 is p. Let point A be a point on line D_1D_2 with $(0,\upsilon,\omega)$ as its coordinate in (μ,υ,ω) system. Then $\hat{\sigma}$, the directional vector of the line integral along $\overrightarrow{SA}$ can be uniquely determined by $\vec{\Phi}$, υ and ω. So the cone-beam projection along line $\overrightarrow{SA}$ can be described by $g(\vec{\Phi},\upsilon,\omega)$. Now Grangeat's formula can be expressed as

$$\frac{\partial}{\partial\rho}R(\hat{\theta},\rho)=\frac{\left|\overrightarrow{SC}\right|^2}{\left|\overrightarrow{SO}\right|^2}\frac{\partial}{\partial p}\int_{-\infty}^{+\infty}\frac{\left|\overrightarrow{SO}\right|^2}{\left|\overrightarrow{SA}\right|^2}g(\vec{\Phi},\upsilon,\omega)dq \tag{5}$$

where $p=\upsilon\cos\alpha+\omega\sin\alpha$, $q=-\upsilon\sin\alpha+\omega\cos\alpha$.
Assume that

$$G\ (\vec{\Phi},\upsilon,\omega)=\frac{\left|\overrightarrow{SO}\right|^2}{\left|\overrightarrow{SA}\right|^2}g(\vec{\Phi},\upsilon,\omega)\ , \tag{6}$$

so we get the new equation from equation (6) as follows:

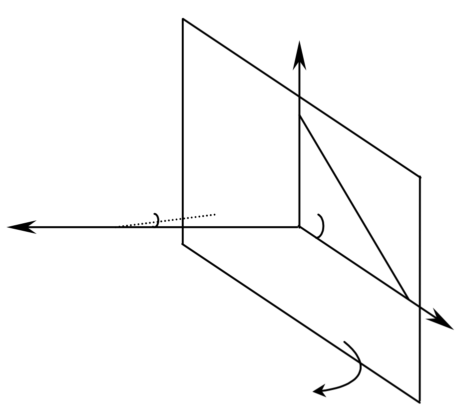
$$\frac{\partial}{\partial\rho}R(\hat{\theta},\rho)=\frac{\left|\overrightarrow{SC}\right|^2}{\left|\overrightarrow{SO}\right|^2}\int_{-\infty}^{+\infty}[\cos\alpha\frac{\partial}{\partial\upsilon}G(\vec{\Phi},\upsilon,\omega)+\sin\alpha\frac{\partial}{\partial\omega}G(\vec{\Phi},\upsilon,\omega)]dq \tag{7}$$

Since the partial derivatives $\frac{\partial}{\partial\upsilon}G(\vec{\Phi},\upsilon,\omega)$ and $\frac{\partial}{\partial\omega}G(\vec{\Phi},\upsilon,\omega)$ need to be calculated only once in the whole reconstruction process, the computational complexity is greatly reduced.

C. Projections Rebinning

The rebinning of projections is to find the transform functions among the parameters on the left side and the right side of equation (8). That is for each point (θ,φ,ρ) in the Radon space, calculating the corresponding S, the focal point position; α, the direction of the integration; and p, the distance from the line of the integration to the origin of the coordinates.

To our volume scanning structure, we adopt the geometry with the positions of detectors and x-ray source reversed. In the case of image reconstruction, the circle and arc, which the detectors are



located, can be regarded as the orbit of cone-beam vertexes; and the trough-like x-ray source can be treated as detector pixels on the trough-like surface. The reconstruction algorithm of orthogonal circle-and-arc orbit volume CT can be utilized in our system. Hence, we adopt the backward mapping method for rebinning projections here [7].

We assume that any Radon plane intersects the circle orbit at two points, named S_1 and S_2, and the position arrangement for $S_1\rightarrow S_2\rightarrow O$ is counterclockwise if one observes from above the circle orbit. Second, let's suppose that the angle between $\overrightarrow{OS_1}$ and the x axis is ϕ_{c1} and that between $\overrightarrow{OS_2}$ and the x axis is ϕ_{c2}. Then, for a given point (θ,φ,ρ) in the Radon space, ϕ_{c1} and ϕ_{c2} can be calculated directly from the coordinates of S_1 and S_2, respectively.

(1) Rebinning from the circular orbit:

For a given (θ,φ,ρ), p and α can be solved exactly for ϕ_{c1}

$$p=\frac{D\rho}{\sqrt{D^2-\rho^2}} \tag{8 a}$$

$$\alpha=\begin{cases}\sin^{-1}(\frac{D\cos\theta}{\sqrt{D^2-\rho^2}}) & \rho\geq 0\\ \pi-\sin^{-1}(\frac{D\cos\theta}{\sqrt{D^2-\rho^2}}) & \rho<0\end{cases} \tag{8 b}$$

and for ϕ_{c2}: $p=\frac{D\rho}{\sqrt{D^2-\rho^2}}$ (9 a)

$$\alpha=\begin{cases}\pi-\sin^{-1}(\frac{D\cos\theta}{\sqrt{D^2-\rho^2}}) & \rho\geq 0\\ \sin^{-1}(\frac{D\cos\theta}{\sqrt{D^2-\rho^2}}) & \rho<0\end{cases} \tag{9 b}$$

(2) Rebinning from the arc orbit:

$$p=\frac{D\rho}{\sqrt{D^2-\rho^2}} \tag{10a}$$

$$\alpha=\begin{cases}\cos^{-1}(\frac{D\sin\theta\sin\varphi}{\sqrt{D^2-\rho^2}}) & \theta\in(0,\pi/2]\\ \pi-\cos^{-1}(\frac{D\sin\theta\sin\varphi}{\sqrt{D^2-\rho^2}}) & \theta\in(-\pi/2,0]\end{cases} \tag{10 b}$$

Next step, we need to find the mapping of the projection data from the plane to the trough-like surface. In Figure 8, for a point $(\vec{\Phi},\alpha,p)$ in the detector plane there is a point $(\vec{\Phi},\beta,t)$ in the trough-like surface, where β and t can be calculated via the following equations:

$$\beta=\pi-\arccos\frac{r_2^2+r_3^2-x^2}{2r_2r_3} \tag{11 a}$$

$$t=\frac{xp\left|\cos\alpha\right|}{\sqrt{(r_2+r_3)^2+p^2\sin^2\alpha}} \tag{11 b}$$

where x is the positive solution of the following equation:

$$r_2^2=x^2+r_3^2-2xr_3\cos(arctg\frac{p\sin\alpha}{r_2r_3}) \tag{12}$$

So far, based on the first derivative of the Radon transform

$\frac{\partial}{\partial \rho} R(\hat{\theta}, \rho)$ from the rebinning process, calculation of the second derivative can be accomplished by convoluting $\frac{\partial}{\partial \rho} R(\hat{\theta}, \rho)$ with a 1-D derivative filter. The object $f(\vec{x})$ can be reconstructed by back projection, as indicated in equation (4).

D. 3-D Reconstruction Algorithm

Once the cone-beam projections are obtained, the procedure to reconstruct the 3D images from the projections is descried as follows:

1) Pre-weighting cone-beam projections, using equation (6);
2) Calculating the partial derivatives of the pre-weighted projections, using equation (7);
3) Rebining the first derivative of Radon transform, using equations (8), (9) and (10);
4) Calculating the second derivative of Radon transform , using equation (11) and (12);
5) Reconstructing the images by inverse Radon transforms, using equation (4).

IV. SIMULATION EXPERIMENT

We make simulation experiment with Shepp-Logan phantom to verify our 3-D imaging system and its reconstruction algorithm. In the experiments, we place point detectors 3° apart on the circle-plus-arc orbit. The total number of detectors is 130. This is equivalent to cone-beam x-ray scanning at 130 positions and gaining 130 groups of projection data. A transverse slice of the Shepp-Logan phantom at z=-38.5 mm is in Figure 5(a) for reference, which is directly calculated from the phantom data. Figure 5(b) is the same slice reconstructed from projection data. Their corresponding profiles are shown in Figure 5 (c) and (d). The artifact in the reconstruction image are due to: (1) sampling rate (the number of detectors on the circle-plus-arc orbit), (2) the error from two phases of rebinning of projection data to Radon space.

V. CONCLUSION

The volume CT with a trough-like source and point-detectors proposed in this paper has some innovations as follows:

(1) A novel geometry of raster scanning x-ray source that simplifies machine structure.

(2) Using certain distributing detectors, the imaging system could gain full projections through the trough-like x-ray source rotation only a single time. So it achieves optimal combination of the following aspects: minimize rotating times, Compton scatter to be restrained, and image reconstruction speed to be improved.

And from above simulation experiments, we can see that the direct volume CT scanning structure proposed in this paper achieves not only faster scanning speed but also more precise reconstruction image than the methods in the literature. Moreover, this structure does not need to move the patient during scanning, which is suitable for image-guided surgery. Besides that, this system specially suits for micro-animal imaging, which can be used to check the micro focus of infection, track the curative effect of new medicine or gene, and so on.

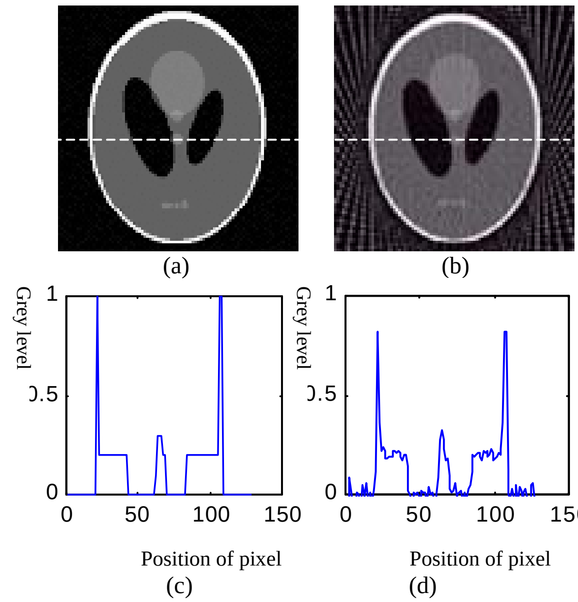


Figure 5. The comparison of original image with reconstruction image at certain position. Figure (a) and (b) are the slice images of Shepp-Logan module and reconstruction image at z=80. Figure (c) and (d) are the profiles at the position indicated by dashed lines in figure (a) and (b).

ACKNOWLEDGMENT

The research work presented in this manuscript was supported by National Natural Science Foundation of China (NO：39870210). The authors wish to thank Prof. M. Yao for proofreading this manuscript.

REFERENCES

[1] H.K.Tuy, “An inversion formula for cone-beam reconstruction,” *SIAM J. Appl. Math*, vol. 43, pp. 546~552, 1983.

[2] H. Kudo, T. Saito. “Derivation and implementation of a cone-beam reconstruction algorithm for nonplanar orbits,” *IEEE Trans. Medical Imaging*, vol. 13, pp. 196~211, January 1994.

[3] P. Oskuoui, Henry Stark. “A comparative study of three reconstruction methods for a limited-view computed tomography problem,” *IEEE Trans. Medical Imaging*, vol. 8, pp. 43~49, January 1989.

[4] K.C.Tam. “Three dimensional computerized tomography scanning method and system for large objects with smaller arc detector,” *U.S. Patent* 5 390 112, 1995.

[5] R. D. Albert, T. M. Albert. “Aerospace applications of x-ray system using reverse geometry,” *Materials Evaluation*, vol. 51, pp. 1350~1352, December 1993.

[6] Xiaohui Wang. “A cone-beam reconstruction algorithm for circle-plus-arc data-acquisition geometry,” *IEEE Trans. Medical Imaging*, vol. 18, pp. 815~824, September 1999.

[7] Y. Weng, G. L. Zeng, and G.T. Gullberg, “A reconstruction algorithm for helical cone-beam SPECT,” *IEEE Trans. Nucl. Sci.*, vol. 40, pp. 1092~1101, August 1993.